Productivity over the Life-Cycle and its Effects on the Interest Rate*

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Abstract

Japan has faced rapid ageing, persistently low interest rates, sluggish growth, and deflation for decades. Concurrently, there has been a gradual convergence in productivity between young and elderly workers. This paper aims to explore the relationship between productivity, demographic shifts, and interest rates in Japan during the post-bubble era, using an overlapping generations two-agent New Keynesian (OTANK) DSGE model. The narrowing productivity gap between younger and older cohorts puts upward pressure on interest rates. Meanwhile, factors such as longer life expectancy and negative population growth rates exert downward pressure on interest rates. The latter effect dominates. A central bank that does not account for this when setting monetary policy may induce deflationary pressure in the economy. Important policy implications emerge: Enhancing worker productivity across workers' entire life-cycle and bridging the productivity gap between younger and older workers can help offset the decline in interest rates, and monetary policy ought to account for shifting demographics.

JEL Codes: E17, J11
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1 Introduction

In the last few decades, Japanese policy makers have faced a unique but increasingly common dimension of heterogeneity: an ageing society. Lise et al. (2014) document a series of striking cross-sectional and life-cycle facts of the Japanese economy, one of which is the demographic shift that the country has undergone: Between 1980 and 2015 the fraction of household heads aged 33 fell from 1.97% to 1.18% while the fraction aged 55 rose from 1.11% to 1.20%.[1] This change in the age composition of the Japanese economy has implications on consumption inequality and the natural interest rate.[2] Empirical studies such as Bailey et al. (2022) and Cesa-Bianchi, Harrison, and Sajedi (2023) have documented that the decline in real interest rates globally are due in significant part to demographic ageing in advanced economies.

In this paper, we investigate the role that the productivity of the elderly plays in the relationship between interest rates and ageing population. Theoretical studies such as Fujiwara and Teranishi (2008) and Carvalho, Ferrero, and Nechio (2016) find that decreases in population growth and an increase in life expectancy together could explain part of the persistent deflation and low interest rates in Japan. They argue that if a central bank fails to take into account these demographic transitions, it may inadvertently compound the problem by setting an overly aggressive monetary policy. We take these transitions as our baseline, and add another demographic change observed in the last decades in Japan: An increase in the relative productivity of elder workers. Wage earnings across different age cohorts have converged, implying that productivity has also converged. Moreover, an increase in life expectancy and healthy life expectancy in the last few decades suggests that the labour productivity of elderly has increased as well. The specifics of the demographic transition are outlined in Section 1.1.

We use an overlapping generations two-agent New Keynesian (OTANK) model based on Yaari (1965), Blanchard (1985), and Gertler (1999) calibrated to Japanese data, to explore the effects of converging productivity across different aged cohorts in an ageing society. Using data from the Japanese Ministry of Health, Labour, and Welfare (MHLW), we find that the relative real wage – which we use as a proxy for relative productivity – of elderly workers to young workers was approximately 0.79 in 1990 and rises to 0.85 in 2017 (see Figure 1).[3] In our model, we assume elder households continue to participate in labour markets, albeit less efficiently than their young worker

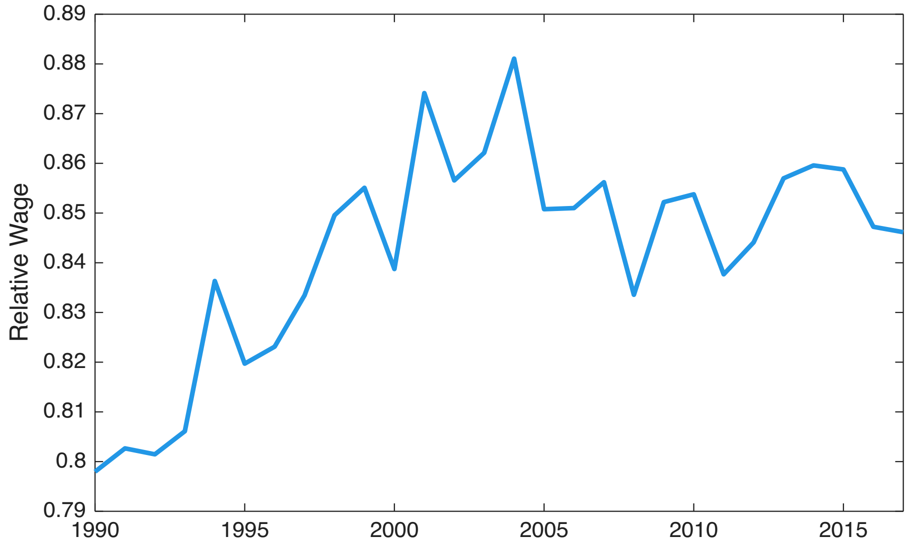
1. See the National Institute of Population and Social Security Research (IPSS) for further details.

2. The secular decline in natural interest rates and real interest rates has garnered attention from policy makers (Bernanke, 2015) due to, for example, its implications regarding asset pricing and slower economic growth (Del Negro et al., 2019).

3. This result is mostly driven by a change in male workers; the relative productivity of female workers is quite stable over the same period.

counterparts.

Figure 1: Relative wage of Japanese elder and young workers



Note: The relative wage of elderly workers, defined as 60 years and older, to young workers, 20 to 59 years, over time. Source: MHLW

We introduce three ways to simulate converging productivity across cohorts: First, we increase the relative productivity of elder workers, but not to parity with young workers. Second, we increase the "retirement age", which is equivalent to increasing the duration at which workers are at full productivity. Third, we combine the first two types of shocks. We find that productivity convergence of the young and the elder workers alleviates the decline of interest rates. When workers are productive for longer, there is less capital accumulation since there is less incentive to save for old age, and therefore an upward pressure on the interest rates. This result may give some credence to the idea that a central bank which does not take demographics into account may be setting an overly-aggressive monetary policy.

Our simulations suggest that downward pressure on the real interest rate induced by ageing demographics leads to a decrease in the nominal interest rate and lower inflation; counter-intuitive to standard macroeconomic analysis of monetary policy and inflation. This neo-Fisherian result is driven by the fact that our agents are in a perfect-foresight environemnt, knowing the full path of the demographic transition as outlined in, for example, García-Schmidt and Woodford (2019), Bilbiie (2022), and Cochrane (2016, 2024).

Finally, we examine the significance of demographics in relation to a series of tax cuts and their ability to stimulate the economy. We find that tax cuts or fiscal deficits are expansionary and self-financed; however, with a declining population a tax cut tends to be less stimulatory and inflationary. These findings are related to the theoretical contributions of Mian, Straub, and Sufi (2022) and Angeletos, Lian, and Wolf (2023) and the fiscal theory of the price level (FTPL) in a sticky-price environment (Cochrane, 2017a, 2017b, 2021). Under the FTPL, sustained large government deficits lead to high inflation in order for government debt to remain sustainable. This channel is subdued in our model which features an overlapping generations environment with population decline.

Relation to the literature. This paper contributes to two strands of macroeconomics. The first is on the relationship between declining interest rates and demographics. The second is on the heterogeneous-agents macroeconomics literature, and a subset of it with overlapping generations models.

In the first strand of literature, the relationship between interest rates and demographics, authors like Braun, Ikeda, and Joines (2009), Carvalho and Ferrero (2014), and Carvalho, Ferrero, and Nechio (2016) establish that an ageing population results in lower real interest rates, because the increased savings increases capital accumulation. Laubach and Williams (2003), and the studies by Bailey et al. (2022), Cesa-Bianchi, Harrison, and Sajedi (2023), and Carvalho et al. (2023) focus on the drivers of the decline in natural interest rates globally and across advanced economies. Hayashi and Prescott (2002), Katagiri (2012), Fujita and Fujiwara (2023), and Chen, İmrohoroğlu, and İmrohoroğlu (2006) focus specifically on Japanese demographics and the decline in real interest rates. In our paper, we explore the productivity channel in the relationship between demographics and real interest rates. We find that the convergence in productivity of young and elder workers partially alleviated the decrease in the real interest rates. In this way, we find potential policy reforms that put upward pressure on interest rates, such as an increase in elder worker productivity and/or an increase in the retirement age.

The second is on the heterogeneous-agents macroeconomics literature, and a subset of it with overlapping generations models. This contemporary literature, established by seminal works such as Gornemann, Kuester, and Nakajima (2016), Bilbiie (2018), Auclert (2019), and Kaplan, Moll, and Violante (2018), focuses on how income and asset distributions of agents in an economy matter for the transmission of monetary policy. Simply said, the key dimension of heterogeneity arises from the different consumption-saving decisions that different (typically "rich" and "poor") households make. This paper, along with Fujiwara and Teranishi (2008), Carvalho and Ferrero (2014), Ikeda and Saito (2014), Sudo and Takizuka (2019), Braun and Ikeda (2021, 2022), and Fujita and Fujiwara (2023), focuses on a different dimension of heterogeneity among agents which is especially relevant for Japan: the elderly and the young. Building on the original life-cycle model presented by Gertler (1999), this paper contributes to the literature explaining the persistent deflation and low interest rates in Japan. We contribute to this strand of literature by using a New Keynesian setup in a overlapping generations model.

The rest of the paper is as follows. Section 1.1 begins by establishing stylised facts in regards to Japanese demographics, productivity, and wages, presenting empirical targets for our quantitative model. Section 2 introduces the OTANK model and explains the model calibration strategy. Section 3 presents the main quantitative experiments conducted in this paper, including our exercises on the ELB and fiscal sustainability. Section 4 concludes the study, and discusses avenues for future research.

1.1 Japanese demographics

The model-relevant Japanese demographic transitions are the following: (i) a decline in the population growth rate, (ii) an increase in life expectancy, and (iii) an increase in the relative productivity of the elderly to the young. The first two are relatively known stylised facts for Japan as an ageing society. The average life expectancy was 78 years in 1990, and is projected to be 88 years by 2050. Despite the increase in life expectancy, the population of Japan has been declining since the early 2010s due to low fertility rates. Where the Japanese population was still growing at 0.43% per year in 1990, the forecast for the rate is -0.57% for 2050.

A lesser known stylised fact relating to the demographic transition in Japan in the past few decades is the convergence of productivity of the elderly to the young. In the model we assume that wages are reflective of productivity, and specifically the marginal product of labour. We analyse average monthly wage earnings data for both Japanese males and females across different age cohorts between 1990 and 2017 from the Japanese Ministry of Health, Labour, and Welfare (MHLW). We calculate the relative wage of the elderly workers, defined as 60 years and older, to young workers, 20 to 59 years, and plot it over time in Figure 1.[4] We define elderly workers as workers over 60 years, the mandatory retirement wage in 1990. Afterwards, firms usually reclassified them as non-regular workers and paid them a lower wage (Kondo, 2016). We can see that since 1990 the relative wage of the elderly has increased substantially. For the purposes of simulations with our quantitative model, we proxy the relative productivity between elderly and young workers with their relative wage.

4. The data shows that earnings over the life-cycle has become more uniform for Japanese males. Japanese females' wage earnings over the lifetime were initially quite flat, suggesting incremental productivity increases over the lifetime. We adjust the plotted data for gender and age shares.

2 The OTANK model

As mentioned previously, the model framework employed is based on Gertler (1999), and extends the works of Fujiwara and Teranishi (2008), Ferrero (2010), and Carvalho and Ferrero (2014). There are three main blocks in the economy: households, firms (including financial firms), and government. Households are divided into two cohorts, young and old, and choose to supply labour in order to fund their consumption and maximise their lifetime utility. Of the income they save, households may choose to invest in three assets: physical capital, domestic government bonds, and/or shares in firms. Firms consist of intermediate goods firms, which produce differentiated goods and are monopolistically competitive, and perfectly competitive final goods firms. The government consists of a fiscal authority and a central bank, which distributes income and sets monetary policy, respectively. The fiscal authority funds its consumption through the issuing of debt and its collection of lump-sum taxes, and it also provides pension funds to the elderly.

2.1 Firms and production

2.1.1 Final good firms

The model features three types of firms which operate in the economy, and their characteristics are standard as in the New Keynesian literature (Christiano, Eichenbaum, and Evans, 2005; Galí, 2015). A continuum of intermediate firms are monopolistically competitive, and combine labour and capital to produce intermediate goods, $Y_t(i)$, where $i \in (0,1)$. Investment goods are produced by perfectly competitive firms. Final good producers, which also operate under perfect competition, produce final goods, Y_t, from intermediate goods, which are then used for consumption, C_t, investment, I_t, and government spending, G_t:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{1-\epsilon}},$$

where $\epsilon > 1$ is a constant elasticity of substitution (CES) parameter for the differentiated intermediate goods. Final good firms maximise their profits by selecting how much of each intermediate good to purchase, and so their problem is:

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di.$$

Thus, as in Blanchard and Kiyotaki (1987), following the first-order condition (FOC) of the final good firm problem, intermediate good producers face a downward sloping demand curve for their products:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t, \tag{1}$$

where $P_t(i)$ is the price of good i and P_t is the price of the final good and also the price index for the aggregate economy and is defined as:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}.$$

2.1.2 Intermediate goods firms

Intermediate firms use aggregate labour, L_t, and capital, K_t, and pay wages, w_t, and rents, r_t^k, as factor payments. Intermediate firm i produces its goods using a constant returns to scale Cobb-Douglas production technology where X_t denotes trend productivity growth, Z_t is a temporary total factor productivity shock, and $\alpha \in (0, 1)$ denotes the labour share of output:

$$Y_t(i) = Z_t \left[K_{t-1}(i)\right]^{\alpha} \left[X_t L_t(i)\right]^{1-\alpha}.$$

Wages, rents, and the real marginal cost (the Lagrangian multiplier from the intermediate firm's problem, φ_t) arises from the first order conditions:[5]

$$\begin{aligned} r_t^k &= \alpha\varphi_t(i) K_{t-1}(i)^{\alpha-1} \left[X_t L_t(i)\right]^{1-\alpha}, \\ w_t &= (1-\alpha)\varphi_t(i) K_{t-1}(i)^{\alpha} \left[X_t L_t(i)\right]^{-\alpha}, \end{aligned}$$

which yields:

$$\varphi_t = \frac{1}{Z_t}\left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left[\frac{w_t}{X_t(1-\alpha)}\right]^{1-\alpha}. \tag{2}$$

Price setting and the NKPC. The next step for the intermediate firm is its dynamic price-setting decision, whereby it solves the following problem:

$$\max_{P_t(i)} \sum_{t=0}^{\infty} \frac{1}{\prod_{s=1}^{t} R_{t+s-1}/\pi_{t+s}} D_t^I(i), \tag{3}$$

5. The cost minimization problem for each intermediate goods producer is:

$$\min_{K_{t-1}(i), L_t(i)} r_t^k K_{t-1}(i) + w_t L_t(i),$$

subject to:

$$Z_t K_{t-1}(i)^{\alpha} \left[X_t L_t(i)\right]^{1-\alpha} \geq Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t.$$

subject to (1), where $D_t^I(i)$ is firm i's per-period profits defined as:[6]

$$D_t^I(i) = \left[\frac{P_t(i)}{P_t} - \varphi_t\right] Y_t(i) - \frac{\phi_I}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 Y_t,$$

and where R_t is the gross nominal interest rate, π_t is gross inflation, ϕ_I is a price adjustment parameter, and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ is the optimal markup charged by intermediate firms arising from monopolistic competition.

The solution to the price-setting problem in a symmetric equilibrium where $P_t(i) = P_t, \forall i$ yields the following New Keynsian Phillips Curve (NKPC):

$$(\pi_t - 1)\pi_t = \frac{(\epsilon - 1)}{\phi_I}\left(\mathcal{M}\varphi_t - 1\right) + \frac{\pi_{t+1}}{R_t}\frac{Y_{t+1}}{Y_t}(\pi_{t+1} - 1)\pi_{t+1}.$$

Finally, because factor payments are standard across all intermediate firms, in the symmetric equilibrium we get:

$$Y_t = Z_t K_{t-1}^{\alpha}(X_t L_t)^{1-\alpha},$$

and the following expenditure share:

$$\frac{w_t L_t}{r_t^K K_{t-1}} = \frac{1-\alpha}{\alpha}.$$

2.1.3 Capital good firms

Capital goods are produced by perfectly competitive firms. The aggregate capital stock grows according to a standard law of motion:

$$K_t = I_t + (1-\delta)K_{t-1},$$

where I_t is investment and $\delta \in (0, 1)$ is the depreciation rate.

The objective of the capital good producing firm is to choose I_t to maximise revenue, $Q_t I_t$. We assume that capital goods producing firms are subject to investment adjustment costs, $\Phi(\cdot)$, as in Christiano, Eichenbaum, and Evans (2005).[7] Thus, the representative capital good producing firm's objective is:

$$\max_{I_t} \sum_{t=0}^{\infty} \frac{D_t^K}{\prod_{s=1}^{t} R_{t+s-1}/\pi_{t+s}},$$

where per-period profits are:

$$D_t^K \equiv \left\{Q_t - 1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right\} I_t.$$

6. For simplification, we assume that intermediate firms discount profits with the real interest rate.

7. We assume the following functional form for $\Phi(\cdot)$:

$$\frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2,$$

with $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(\cdot) > 0$.

Solving the optimisation problem yields the following first-order condition for the price of capital:[8]

$$Q_t = 1 + \Phi\left(\frac{I_t}{I_{t-1}}\right) + \Phi'\left(\frac{I_t}{I_{t-1}}\right) - \frac{\pi_{t+1}}{R_t}\Phi'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2.$$

2.1.4 Financial intermediaries

Finally, for the supply side of the economy, we assume that perfectly competitive mutual funds which trade claims on capital rental income, government bonds, and equity, packaging it into risk-free financial assets sold to households. More specifically, mutual funds investment in government bonds B_t, physical capital,[9] and equity of intermediate good producing firms $x_{I,t}$. Government bonds pay a nominal risk-free rate R_t, and intermediate good firm shares are priced at P_t^I and pay a dividend of D_t^I. Meanwhile, mutual funds sell a composite financial asset to households A_t which pay a nominal one-period, non-state contingent return R_t^M. As such, the representative mutual fund's objective function is:

$$\max_{\{A_t, B_t/P_t, K_t, x_{I,t}\}} \sum_{t=0}^{\infty} \frac{\Pi_t^M}{\prod_{s=1}^{t} R_{t+s-1}/\pi_{t+s}},$$

where per-period profits of the mutual fund are:

$$\Pi_t^M = A_t + \frac{R_{t-1}}{\pi_t}\frac{B_{t-1}}{P_{t-1}} + (r_t^K + (1-\delta)Q_t)K_{t-1} + (P_t^I + D_t^I)x_{I,t-1} - \frac{R_{t-1}^M}{\pi_t}A_{t-1} - \frac{B_t}{P_t} - Q_t K_t - P_t^I x_{I,t}.$$

We thus get the following no-arbitrage condition across the different financial assets in the economy:

$$\frac{R_t}{\pi_{t+1}} = \frac{R_t^M}{\pi_{t+1}} = \frac{r_{t+1}^K + (1-\delta)Q_{t+1}}{Q_t} = \frac{P_{t+1}^I + D_{t+1}^I}{P_t^I}.$$

Therefore, for simplicity, throughout the rest of the paper we assume that $R_t = R_t^M$.

2.2 Households

As in Gertler (1999) and corresponding overlapping generations models, households have finite lives and go through two stages in life: youth and old age. In order to develop a parsimonious model with tractable consumption/savings behaviour and realistic lifetimes, simplifying assumptions are made pertaining to population dynamics, insurance markets, and household preferences.

As such, labour supply and the population mass of households differ across cohorts. Consider the period $t-1$, and let the population mass of the young and old

8. Note that here we do not adjust for trend. Appendix B provides a full set of de-trended equilibrium conditions.

9. Alternatively, one can think of these as investment goods.

be denoted as N^y_{t-1} and N^e_{t-1}, respectively. Between $t-1$ and t, a young household remains young with probability ω_t, and with probability $1-\omega_t$ becomes old. Thus, the average length of time an individual spends in her youth is given by $\frac{1}{1-\omega_t}$. In addition, an elderly individual survives from $t-1$ to t with probability γ_t, and conversely perishes with probability $1-\gamma_t$. In period t, $(1-\omega_t+n_t)N^y_{t-1}$ new young households are born, giving the following law of motion for the young population:

$$\begin{aligned}N^y_t &= (1-\omega_t+n_t)N^y_{t-1}+\omega_t N^y_{t-1}\\ &= (1+n_t)N^y_{t-1}.\end{aligned}$$

So, n_t is the growth rate of the labour force between periods $t-1$ and t. Meanwhile, the law of motion for the elderly population is given as:

$$N^e_t = (1-\omega_t)N^y_{t-1}+\gamma_t N^e_{t-1}.$$

As will be discussed, the young and elderly possess different consumption, saving, and asset profiles, therefore we can track the dependency ratio, $\Gamma_t = N^e_t/N^y_t$, defined as:

$$\Gamma_t = \frac{(1-\omega_t)N^y_{t-1}+\gamma_t N^e_{t-1}}{(1+n_t)N^y_{t-1}},$$

with the following law of motion:

$$(1+n_t)\Gamma_t = (1-\omega_t)+\gamma_t\Gamma_{t-1}.$$

For simplification, we assume no aggregate risk for households. Risk is idiosyncratic for both the young and old: the young may suddenly face a drop in productivity, and the elderly face an uncertain time of death. To counter these uncertainties we assume a perfect insurance market, as in Yaari (1965) and Blanchard (1985). The elderly purchase annuities in each period from the mutual funds. In each following period, γ_t fraction of elderly that survive receive all the returns – there is no estate bequest mechanism for the $1-\gamma_t$ fraction of elderly that perish. As such, if R_t is the gross return on investments made by the mutual fund, then the gross return on wealth for the elderly that survive is given by R_t/γ_t.

To account for the loss in productivity (and income) of young households that become old, the model features a special class of recursive preferences (Kreps and Porteus, 1978; Epstein and Zin, 1989) that assumes risk neutrality. We employ these preferences since we can easily obtain analytical optimality conditions since transition probabilities ω_t and γ_t are independent of an agent's age or retirement status, respectively. Also, it allows us to separate risk aversion from intertemporal substitution. These preferences restrict agents to being risk neutral with respect to income risk, but with arbitrary intertemporal elasticity of substition (IES) (Farmer, 1990; Gertler, 1999). Income risk is merely a product of the exogenous "retirement" probability $(1-\omega_t)$, and so the impact of this income risk is mitigated by assuming risk neutrality. With standard Von-Neumann-Morgenstern preferences, young workers would exhibit too much precautionary savings with respect to income risk. We can then pick the curvature parameter ρ, which pins down the IES $\sigma = (1-\rho)^{-1}$, allowing us to broadly match empirical responses of consumption and savings to interest rate changes.

Additionally, there is no insurance market which covers the risk of income loss of a young household. The primary reason for this is to capture life-cycle behaviour. The existence of a perfect insurance market would allow an individual to perfectly smooth their income between being young and old. Thus, in the absence of such an insurance market, income earning potential is mostly skewed towards individuals during their working life. This provides a better representation of the life-cycle.

Let V_t^z denote an individual's recursive utility where the superscript $z = \{y, e\}$ indicates whether the individual is young (y) or elderly (e). Individuals maximise utility by choosing consumption, C_t^z, labour supply, L_t^z, and their asset allocations (described below). Let β^z be an individual's subjective discount factor. Preferences are given by:[10]

$$V_t^z = \left\{ \left[(C_t^z)^{\upsilon} (1 - L_t^z)^{1-\upsilon} \right]^{\rho} + \beta_{t+1}^z (V_{t+1}|z)^{\rho} \right\}^{\frac{1}{\rho}}, \tag{4}$$

where:

$$\beta_{t+1}^y = \beta,$$
$$\beta_{t+1}^e = \gamma_{t+1}\beta,$$

as the young and elderly have different discount factors due to the risk of death. Additionally, ρ pins down the intertemporal elasticity of substitution and υ is an individual's utility weight on consumption.[11]

2.2.1 Elder workers

An elderly individual born in period j and enters retirement age in period k chooses consumption, $C_t^e(j,k)$, labour supply, $L_t^e(j,k)$, and assets, $A_t^e(j,k)$, comprising of bonds and shares, to maximise their utility given by:

$$V_t^e(j,k) = \max_{C_t^e(j,k), L_t^e(j,k), A_t^e(j,k)} \left\{ \left[C_t^e(j,k)^{\upsilon} (1 - L_t^e(j,k))^{1-\upsilon} \right]^{\rho} + \beta\gamma_{t+1} V_{t+1}^e(j,k)^{\rho} \right\}^{\frac{1}{\rho}}, \tag{5}$$

10. Note that we omit expectation operators since we only consider a perfect-foresight setting.

11. An individual's future value of utility in (4) differs between either being young or old due to the probability of transitioning between the two life stages:

$$V_{t+1}^y = \omega_{t+1} V_{t+1}^y + (1 - \omega_{t+1}) V_t^e.$$

subject to

$$C^e_t(j,k) + A^e_t(j,k) = \frac{1}{\gamma_t}\frac{R_{t-1}}{\pi_t}A^e_{t-1}(j,k) + \varsigma_t w_t L^e_t(j,k) + E^e_t(j,k), \tag{6}$$

where $\varsigma_t = [0,1]$ denotes the relative productivity of an elderly worker to a young worker, and $E^e_t(j,k)$ denotes social security transfers. For model tractability we require that an elderly individual's initial asset holdings be equal to its last period asset holdings during youth:

$$A^e_k(j,k) = A^y_{k-1}(j).$$

From the first order conditions, which can be found in the Appendix A.1, a relationship between labour supply and consumption can be derived:

$$L^e_t(j,k) = 1 - \frac{1-\upsilon}{\upsilon}\frac{1}{\varsigma_t w_t}C^e_t(j,k), \tag{7}$$

as can the consumption Euler equation:

$$C^e_{t+1}(j,k) = \left(\beta\frac{R_t}{\pi_{t+1}}\right)^{\sigma}\left(\frac{w_t}{w_{t+1}}\right)^{\rho(1-\upsilon)\sigma} C^e_t(j,k), \tag{8}$$

where $\sigma = 1/(1-\rho)$.

A recursive expression for non-financial wealth of an elderly individual can be written as the following:

$$H^e_t(j,k) = \varsigma_t w_t L^e_t(j,k) + \frac{\pi_{t+1}}{R_t}\gamma_{t+1}H^e_{t+1}(j,k). \tag{9}$$

We can also write the present discounted value of social security benefits as:

$$S^e_t(j,k) = E^e_t(j,k) + \frac{\pi_{t+1}}{R_t}\gamma_{t+1}S^e_{t+1}(j,k). \tag{10}$$

From these equations, we can guess that the consumption function for an elderly individual is a fraction of total wealth:

$$C^e_t(j,k) = \xi^e_t\left[\frac{R_{t-1}}{\pi_t}\frac{A^e_{t-1}(j,k)}{\gamma_t} + H^e_t(j,k) + S^e_t(j,k)\right], \tag{11}$$

where ξ^e_t is the marginal propensity to consume (MPC) for an elderly individual which satisfies the following first order non-linear difference equation:

$$\frac{1}{\xi^e_t} = 1 + \gamma_{t+1}\beta^{\sigma}\left(\frac{R_t}{\pi_{t+1}}\right)^{\sigma-1}\left(\frac{w_t}{w_{t+1}}\right)^{\rho(1-\upsilon)\sigma}\frac{1}{\xi^e_{t+1}}. \tag{12}$$

Notice that ξ^e_t is independent of any individual elderly worker. This is crucial when it comes to aggregation later. Finally, an expression for the value function satisfying the above equations can be found:

$$V^e_t(j,k) = \left(\xi^e_t\right)^{-\frac{1}{\rho}} C^e_t(j,k)\left(\frac{1-\upsilon}{\upsilon}\frac{1}{\varsigma_t w_t}\right)^{1-\upsilon}. \tag{13}$$

2.2.2 Young workers

A young individual born in period j with no initial assets chooses consumption, $C_t^y(j)$, labour supply, $L_t^y(j)$, and assets, $A_t^y(j)$, to maximise their utility:

$$V_t^y(j) = \max_{C_t^y(j), L_t^y(j), A_t^y(j)} \left\{ \begin{array}{c} \left(C_t^y(j)^\upsilon \left[1 - L_t^y(j)\right]^{1-\upsilon}\right)^\rho \\ +\beta \left[\omega_{t+1} V_{t+1}^y(j) + (1-\omega_{t+1}) V_{t+1}^e(j, t+1)\right]^\rho \end{array} \right\}^{\frac{1}{\rho}}, \tag{14}$$

subject to

$$C_t^y(j) + A_t^y(j) + T_t^y(j) = \frac{R_{t-1}}{\pi_t} A_{t-1}^y(j) + w_t L_t^y(j), \tag{15}$$

where $T_t^y(j)$ are pension payments paid by young individuals.

From the first order conditions, which can be found in the Appendix A.2, the optimal relationship between consumption and labour supply is:

$$L_t^y(j) = 1 - \frac{1-\upsilon}{\upsilon} \frac{1}{w_t} C_t^y(j). \tag{16}$$

In line with equation (13), we can conjecture a value function for a young individual as:

$$V_t^y(j) = (\xi_t^y)^{-\frac{1}{\rho}} C_t^y(j) \left(\frac{1-\upsilon}{\upsilon} \frac{1}{w_t}\right)^{1-\upsilon}. \tag{17}$$

We can then combine this conjectured equation with (13) and the first order conditions yields the following consumption Euler equation for a young individual:

$$\begin{aligned} &C_t^y(j) \left[\frac{\beta R_t \Omega_{t+1}}{\pi_{t+1}} \left(\frac{w_t}{w_{t+1}}\right)^{\rho(1-\upsilon)}\right]^\sigma \\ &\quad = \omega_{t+1} C_{t+1}^y(j) + (1-\omega_{t+1}) \Xi_{t+1}^{\frac{\sigma}{1-\sigma}} C_{t+1}^e(j, t+1) \left(\frac{1}{\varsigma_t}\right)^{1-\upsilon}, \end{aligned} \tag{18}$$

where Ξ_t is the ratio of MPCs of the old and young:

$$\Xi_t = \frac{\xi_t^e}{\xi_t^y},$$

and the adjustment factor to account for the different MPCs between the old and young is given as:

$$\Omega_t = \omega_t + (1-\omega_t) \Xi_t^{\frac{1}{1-\sigma}} \left(\frac{1}{\varsigma_t}\right)^{1-\upsilon}. \tag{19}$$

The present values of non-financial wealth and social security, respectively, are:

$$\begin{aligned} H_t^y(j) &= w_t L_t^y(j) + \omega_{t+1}\frac{\pi_{t+1}}{R_t\Omega_{t+1}}H_{t+1}^y(j) \\ &\quad + (1-\omega_{t+1})\Xi_{t+1}^{\frac{1}{1-\sigma}}\left(\frac{1}{\varsigma_t}\right)^{1-\upsilon}\frac{\pi_{t+1}}{R_t\Omega_{t+1}}H_{t+1}^e(j,t+1), \end{aligned} \tag{20}$$

$$\begin{aligned} S_t^y(j) &= \omega_{t+1}\frac{\pi_{t+1}}{R_t\Omega_{t+1}}S_{t+1}^y(j) - T_t^y \\ &\quad + (1-\omega_{t+1})\Xi_{t+1}^{\frac{1}{1-\sigma}}\left(\frac{1}{\varsigma_t}\right)^{1-\upsilon}\frac{\pi_{t+1}}{R_t\Omega_{t+1}}S_{t+1}^e(j,t+1). \end{aligned} \tag{21}$$

We then conjecture a consumption function for young workers:

$$C_t^y = \xi_t^y\left[\frac{R_{t-1}}{\pi_t}A_{t-1}^y(j) + H_t^y(j) + S_t^y(j)\right]. \tag{22}$$

Then, use the consumption Euler equation (18), conjectured consumption function (22), and the per-period budget constraint (15), to yield an expression for the young worker's MPC:

$$\frac{1}{\xi_t^y} = 1 + \beta^\sigma\left(\frac{R_t\Omega_{t+1}}{\pi_{t+1}}\right)^{\sigma-1}\left(\frac{w_t}{w_{t+1}}\right)^{\rho(1-\upsilon)\sigma}\frac{1}{\xi_{t+1}^y}. \tag{23}$$

2.2.3 Household aggregation

As is evident from Equations (12), (19), and (23), the MPCs of young and old workers are independent of individual characteristics. Combine this with the facts that consumption and labour supply decisions are linear, and that taxes and pension benefits are lump-sum by nature,[12] and we can write the following aggregate consumption functions:

$$C_t^y = \xi_t^y\left(\frac{R_{t-1}}{\pi_t}A_{t-1}^y + H_t^y + S_t^y\right), \tag{24}$$

$$C_t^e = \xi_t^e\left(\frac{R_{t-1}}{\pi_t}A_{t-1}^e + H_t^e + S_t^e\right), \tag{25}$$

for the young and old workers, respectively. It follows that we can also write aggregate labour supply for the two cohorts as:

$$L_t^y = N_t^y - \frac{1-\upsilon}{\upsilon}\frac{1}{w_t}C_t^y, \tag{26}$$

$$L_t^e = \Gamma_t N_t^y - \frac{1-\upsilon}{\upsilon}\frac{1}{\varsigma_t w_t}C_t^e. \tag{27}$$

Aggregate non-financial wealth for the two cohorts needs to be adjusted to account for the population growth rate. Thus, they can be expressed as (with some slight alge-

12. In other words, taxes paid by young workers and the pension benefits for the elderly are independent of an individual's demographic characteristics.

braic simplification):

$$H_t^y = w_t L_t^y + \frac{\omega_{t+1}}{(1+n_{t+1})} \frac{\pi_{t+1}}{R_t \Omega_{t+1}} H_{t+1}^y + \frac{(1-\omega_{t+1})}{(1+n_{t+1})} \Xi_{t+1}^{\frac{1}{1-\sigma}} \left(\frac{1}{\varsigma_t}\right)^{1-\upsilon} \frac{\pi_{t+1}}{R_t \Omega_{t+1}} H_{t+1}^e,$$

$$H_t^e = \varsigma_t w_t L_t^e + \frac{\gamma_{t+1}}{(1+n_{t+1})} \frac{\pi_{t+1}}{R_t} H_{t+1}^e.$$

Likewise, aggregate social security for the young and elderly can be expressed respectively as:

$$S_t^y = \frac{\omega_{t+1}}{(1+n_{t+1})} \frac{\pi_{t+1}}{R_t \Omega_{t+1}} S_{t+1}^y + \frac{(1-\omega_{t+1})}{(1+n_{t+1})} \Xi_{t+1}^{\frac{1}{1-\sigma}} \left(\frac{1}{\varsigma_t}\right)^{1-\upsilon} \frac{\pi_{t+1}}{R_t \Omega_{t+1}} S_{t+1}^e - T_t^y,$$

$$S_t^e = E_t^e + \frac{\gamma_{t+1}}{(1+n_{t+1})} \frac{\pi_{t+1}}{R_t} S_{t+1}^e.$$

To account for the heterogeneity across the two cohorts, an additional state variable is needed to represent the proportion of asset holdings held by either cohort. To this end, let Ψ_t be the proportion of assets held by the elderly:

$$\Psi_t = \frac{A_t^e}{A_t}.$$

The aggregate consumption is then given by:

$$C_t = \xi_t^y \left[(1-\Psi_{t-1}) \frac{R_{t-1} A_{t-1}}{\pi_t} + H_t^y + S_t^y\right] + \xi_t^e \left[\Psi_{t-1} \frac{R_{t-1} A_{t-1}}{\pi_t} + H_t^e + S_t^e\right].$$

We can also characterise the law of motion of aggregate asset holdings across the two cohorts as:

$$A_t^y = \omega_{t+1} \left(\frac{R_{t-1}}{\pi_t} A_{t-1}^y + w_t L_t^y - C_t^y - T_t^y\right), \tag{28}$$

$$\begin{aligned} A_t^e = &\frac{R_{t-1}}{\pi_t} A_{t-1}^e + \varsigma_t w_t L_t^e + E_t^e - C_t^e \\ &+ (1-\omega_{t+1}) \left(\frac{R_{t-1}}{\pi_t} A_{t-1}^y + w_t L_t^y - C_t^y - T_t^y\right). \end{aligned} \tag{29}$$

Households hold the entirety of all assets in the economy:

$$A_t \equiv A_t^y + A_t^e,$$

and thus, by substituting (25) and (28) into (29), the law of motion of the distribution of financial wealth, Ψ_t, evolves according to:

$$\begin{aligned} &[\Psi_t - (1-\omega_{t+1})] A_t = \\ &\omega_{t+1} \left[(1-\xi_t^e) \frac{R_{t-1}}{\pi_t} \Psi_{t-1} A_{t-1} + \varsigma_t w_t L_t^e + E_t^e - \xi_t^e \left(H_t^e + S_t^e\right)\right]. \end{aligned}$$

2.3 Fiscal and monetary policy

Fiscal and monetary policy is undertaken by the government and central bank, respectively. The government collects lump-sum taxes from the young population and issues nominal debt, $P_t B_t$, to finance government spending, G_t, and pension expenses. The budget government constraint, in real terms, is given as:

$$\frac{R_{t-1}}{\pi_t}\frac{B_{t-1}}{P_{t-1}} + E_t + G_t = \frac{B_t}{P_t} + T_t. \tag{30}$$

The government adheres to the following rules pertaining to issuing debt and expenditures:

$$\frac{G_t}{Y_t} = S_t^g, \tag{31}$$

$$\frac{B_t/P_t}{Y_t} = S_t^b, \tag{32}$$

$$E_t = E_t^e, \tag{33}$$

$$T_t = T_t^y. \tag{34}$$

As in Carvalho, Ferrero, and Nechio (2016), the government offers social security benefits to elderly workers:

$$E_t = \varrho_t \left(w_t L_t^y - T_t\right),$$

where $\varrho_t \in [0, 1]$ is the net replacement rate.

Monetary policy is set according to the following inflation-targeting rule:

$$R_t = \bar{R}^{\phi_R}(R_t^n)^{1-\phi_R}\pi_t^{\phi_\pi}, \tag{35}$$

where $\bar{R}$ is the steady state nominal interest rate, R_t^n is the flexible price equilibrium real interest rate, and $\phi_R \in [0, 1]$.

2.4 Market clearing

Market clearing conditions are provided by the following equations pertaining to the intermediate goods market, final goods market, capital market, asset market, and labour market. The economy wide resource constraint is given by:

$$Y_t = C_t + \left[1 + \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + G_t + \frac{\phi_I}{2}(\pi_t - 1)^2 Y_t. \tag{36}$$

In aggregate, financial assets in the economy must equal the sum of capital, bonds, and total shares in intermediate firms:[13]

$$A_t = B_t + K_t + P_t^I.$$

Finally, aggregate labour is the sum of the variable labour supplied by both the young

13. Total net supply of shares in intermediate firms is set to one (in other words, $x_t^I = 1$).

and old:

$$L_t = L_t^y + \varsigma_t L_t^e. \tag{37}$$

2.5 Equilibrium

An equilibrium system of equations is such that final good producers maximise profits subject to their resource constraint (36); intermediate good producers maximise profits (3) taking demand for their differentiated goods as given; households, both young and old, maximise utility (4) subject to their budget constraints ((6) and (15)), taking prices and wages as given; when the government chooses debt and taxes to satisfy its budget constraint (30); when the central bank sets nominal interest rates based on its interest rate rule (35); and when the market for goods and labour clears ((36) and (37)).

Data based on real world observations and projections are input into the model for it to have realistic demographic transitions. In particular, shocks fed into the model are based on changes in ω, γ, n, and ς, and they drive the model's dynamics. Agents have perfect foresight of the full transition path of the economy. Furthermore, all model variables are adjusted for productivity and population growth, and are defined in per-capita terms. The complete set of first-order conditions of the model are outlined in Appendix B.

2.6 Parameterisation

Parameter values for the baseline model are given in Table 1. Each period in the model is one quarter, and so parameters are given in quarterly values unless otherwise stated. As the paper is primarily focused on describing data facts since the collapse of the asset price bubble, the model targets parameter values corresponding to the period between 1990 and 2017.

An individual enters the model economy as a 20 year old young worker, and on average transitions to an elderly worker at the age of 60, in line with the mandatory retirement age in 1990. After this age, firms usually classify as non-regular workers and pay them a lower wage (Kondo, 2016). In line with data and forecasts from the United Nations (UN) *World Population Prospects: The 2017 Revision*, in 1990 individuals are expected to have an average lifetime of 78 years and the population annual growth rate is 0.43%. By 2050, an average Japanese individual is expected to live until the age of 88, and the population growth rate is forecast to be -0.57% per annum. These figures discipline our choices for n and γ, and recall that γ is the survival probability once an individual transitions from young to old. Thus they live for $1/(1-\gamma)$ periods as an

Table 1: **Parameter values and exogenous variables**

Parameter	Value	Description
ω	$[0.9944, 0.9950]$	Transition from young to old
γ	$[0.9815, 0.9860]$	Survival probability
n	$[0.001075, -0.001425]$	Population growth rate
ς_t	$[0.75, 0.85]$	Young-old relative productivity
x	0.0014	Productivity growth rate
α	0.377	Labour share of income
δ	0.025	Depreciation rate
κ_I	2/3	Investment adjust cost term
ϵ	12	Price elasticity of demand
ϕ_I	132	Rotemberg pricing parameter
ρ	-1	Pins down σ
σ	$(1-\rho)^{-1}$	Intertemporal elasticity of substitution
υ	0.70	Preference weight of consumption over leisure
g	0.17	Government spending (fraction of GDP)
b	2	Government debt (fraction of yearly GDP)
ϱ	0.4	Net replacement rate
ϕ_π	1.5	Monetary policy response to inflation
ϕ_R	0.33	Monetary policy weight
β	1.005	Discount factor (real interest rate target)

elder worker. Thus, under the baseline calibration and in 1990, an individual enters the workforce at 20 years of age, works as a young worker until they are 60, retires, and has an expected 18 years of life as an elder worker. So, in the model the effective age of labour market exit coincides with the life expectancy.

We calibrate ς, the relative productivity between young and elderly workers to 0.75 in 1990 for the baseline simulation of the model. For Simulations 2 and 4, ς increases following a concave, monotonically increasing process to 0.85 in 2015, based on real wage data from the MHLW. We assume that ς is then fixed at 0.85 until the end of the simulation period.

The remaining parameters are set according to common values in the literature (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007). The growth rate of labour augmenting productivity, x_t, is an updated version of Hayashi and Prescott (2002). This corresponds to a quarterly growth rate of 0.14% (or 0.56% per annum). The labour share of income, α, is set to 0.377 as per Braun et al. (2006). The capital depreciation rate, δ, is set to 2.5% per quarter, and the investment adjustment cost parameter κ_I is set to 2/3 reflecting the empirical findings of Eberly (1997).

The elasticity of substitution between intermediate goods, κ, is set to 12 in order to generate a steady state markup of 9% (Høj et al., 2007). The Rotemberg pricing parameter, ϕ_I, is set to 132 so that prices have an average duration of four quarters, which broadly fits Japanese data according to Higo and Saita (2007).[14] The IES, σ, is set to 0.5, slightly higher than the baseline 0.25 setting in Gertler (1999), and in accordance to empirical evidence from Hall (1988) and Yogo (2004). We calibrate household preferences of consumption over leisure $v = 0.7$, slightly higher than in Gertler (1999), in order to target labour supply of both young and elderly workers to the findings in Cooley and Prescott (1995) and Boppart and Krusell (2020).

Government consumption as a percentage of GDP, g, is set to 0.17, an averaged value between 1990 and 2016 according to the World Bank and OECD national accounts. It should be noted that we use net debt as a parameter target, and that government debt shows large differences between net and gross debt. However, due to concerns over overvalued assets on government balance sheets, a simple average of both gross and net government debt as a fraction of GDP from the IMF World Economic database is taken as the parameter target for b. The net replacement rate ϱ, which determines social security benefits in the model, is set to 0.3. This is slightly below the evidence from Yashiro and Oshio (2008), but is set in line with the 2022 release of the OECD "Pensions at a Glance Indicators".

For the central bank monetary policy rule, in the baseline simulations we assume $\phi_\pi = 1.5$ and $\phi_R = 0.33$. In subsequent simulations and counterfactuals, we alter these values but make clear our assumptions. Finally, the discount factor β is calibrated to 1.005 to target the initial real interest rate in 1990 of approximately 5-6% per annum.

3 Model simulations

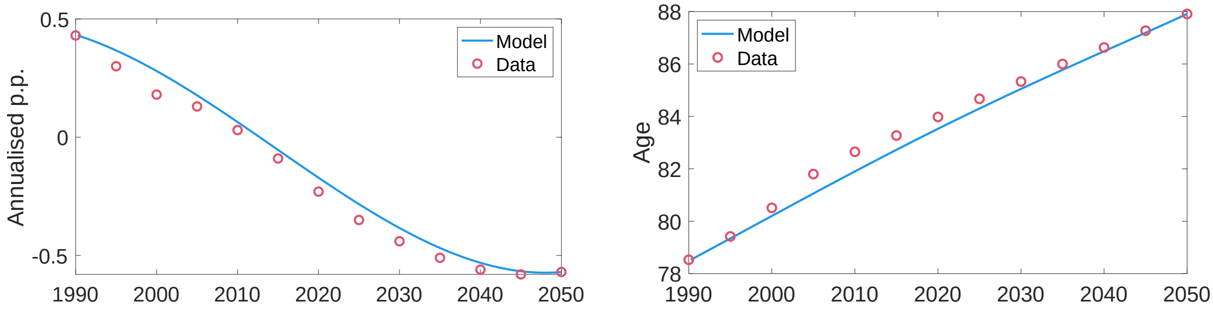
In this section we show the results of our numerical analysis with the model as described above. First we show the effect of the demographic transition in Japan on the real interest rate. We show that a decrease in population growth rates and an increase in life expectancy can explain the decrease in real interest rates in Japan. We also perform counterfactual policy exercises to counteract the decline in real interest rates. Second, we present our analysis on the interaction of the demographic transition and the ELB. In our final simulation, we then test efficacy of fiscal stimulus with and without a demographic transition.

3.1 The effects of demographic transition

There are four types of demographic shifts that our paper focuses on: A decrease in the population growth rate, an increase in life expectancy, an increase in elder worker

14. With this specification, the linearised NKPC slope is equivalent to a staggered pricing model à la Calvo with an average price change every four quarters.

Figure 2: Fitted shocks to population growth rate and life expectancy



(a) Population growth rates (b) Life expectancy

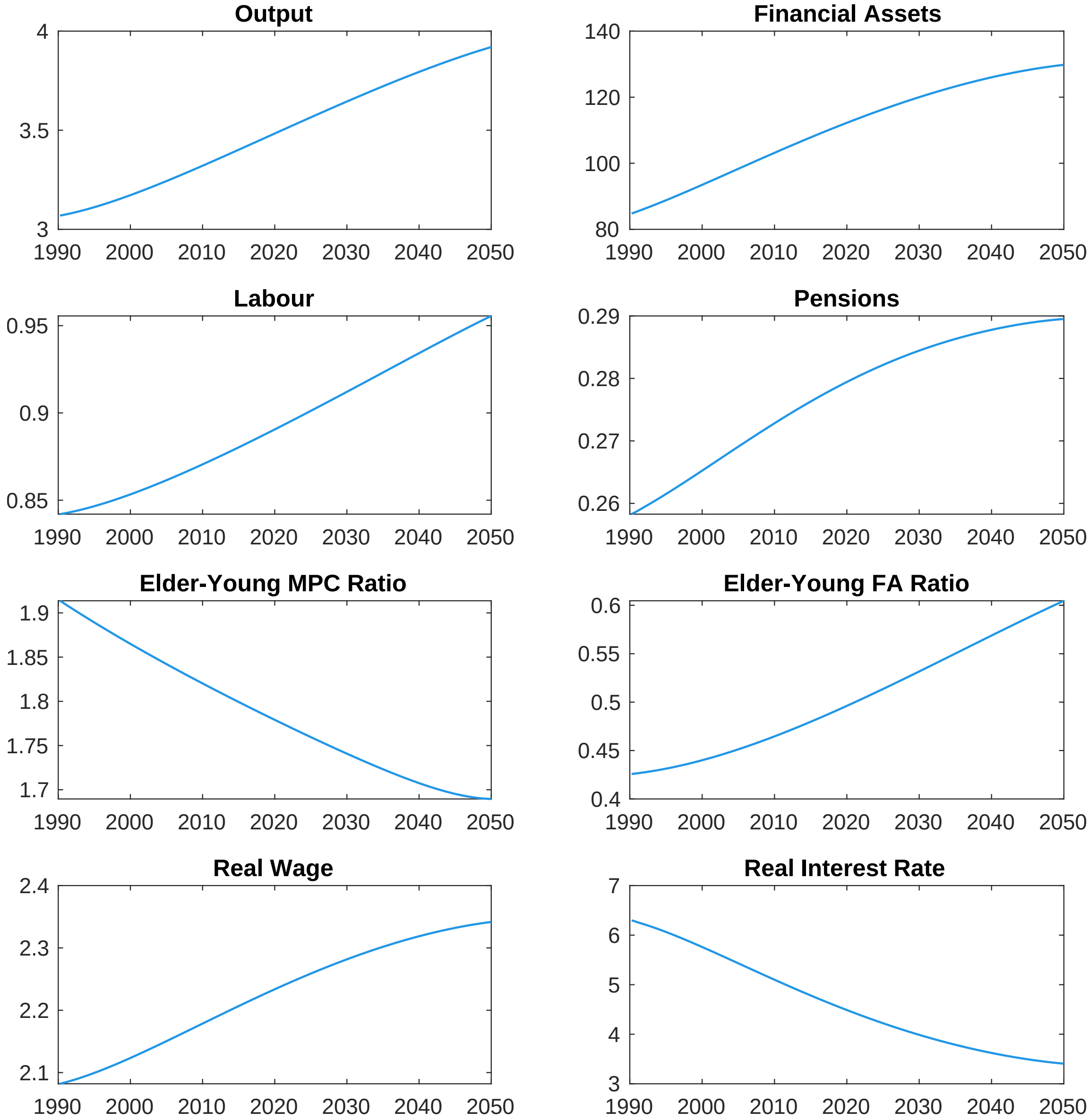
Source: UN *World Population Prospects: The 2017 Revision*. Data points after 2015 are forecasts.

productivity, and an increase in the retirement age. Our baseline scenario (Simulation 1) uses the first two shocks and shows that these exert downward pressure on the real interest rate. Then, we add the increase in elder worker productivity and delay in retirement age as a policy counterfactual, first separately (Simulation 2 and 3) and then together (Simulation 4), to show that those alleviate the downward pressure exerted on real interest rates by the demographic processes from the baseline scenario.

Simulation 1 (baseline): A decrease in population growth rate and an increase in life expectancy. We feed in the decrease in population growth rates and an increase in life expectancy using a smoothing process according to data and projections as shown in Figure 2. The population growth rate was about 0.4% in 1990 and is projected to be almost -0.6% in 2050. The life expectancy was around 78 years in 1990 and is projected to be 88 years in 2050 (an average of both males and females).

We simulate the model by feeding these shocks into the model and plot the time path of key model variables in Figure 3. The decrease in population growth and the increase in life expectancy have the following effect on the economy: Because there are more elder workers in the economy, pensions expenditures increase. Moreover, financial assets increase through higher aggregate savings because agents anticipate living longer and funding consumption during retirement in which their productivity – and thus wage – is relatively low compared to their youth. The financial assets ratio between the elderly and the young increases too: First, there are more elderly workers due to the higher life expectancy, and they also accumulate financial assets. Second, the dependency ratio increases (there are relatively less young people) from the decline in population growth. The economy-wide increase in financial assets decreases the (real) interest rate, as we see in the case for Japan empirically – we expand on this point further below.

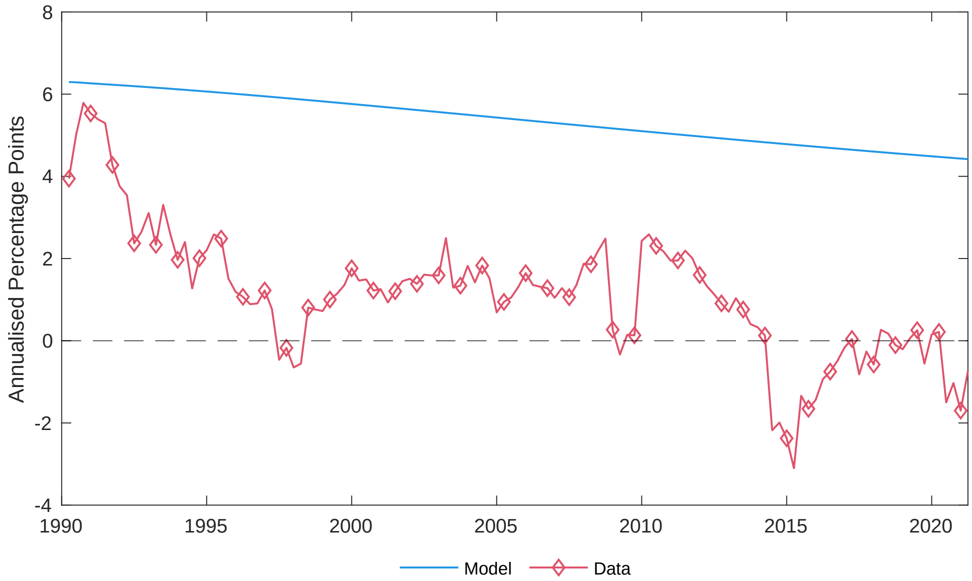
Figure 3: Simulation 1 (baseline; decline in population growth rate and increase in life expectancy)



Note: Figure plots response of variables as n transitions from 0.0011 to -0.0014 and γ transitions from 0.9815 to 0.9860. Interest rate is expressed in net annualised percentage points.

Aggregate labour supply increases as a result of the two shocks, which indicates that the effect of the increase in labour from a higher life expectancy exceeds the effect of a decrease in population growth rate. Agents also work more hours because they engage in precautionary savings for their retirement. This is due to the fact that labour supply and capital accumulation increases, output also increases. The marginal propensity to consume (MPC) is inversely related to the interest rates. So, the de-

Figure 4: Model vs data: Japanese short-term real interest rate



Note: The real interest rate is constructed using the "Overnight Call Money/Interbank Rate" net of inflation using the GDP deflator. Data sourced from FRED, IMF International Financial Statistics, and the OECD.

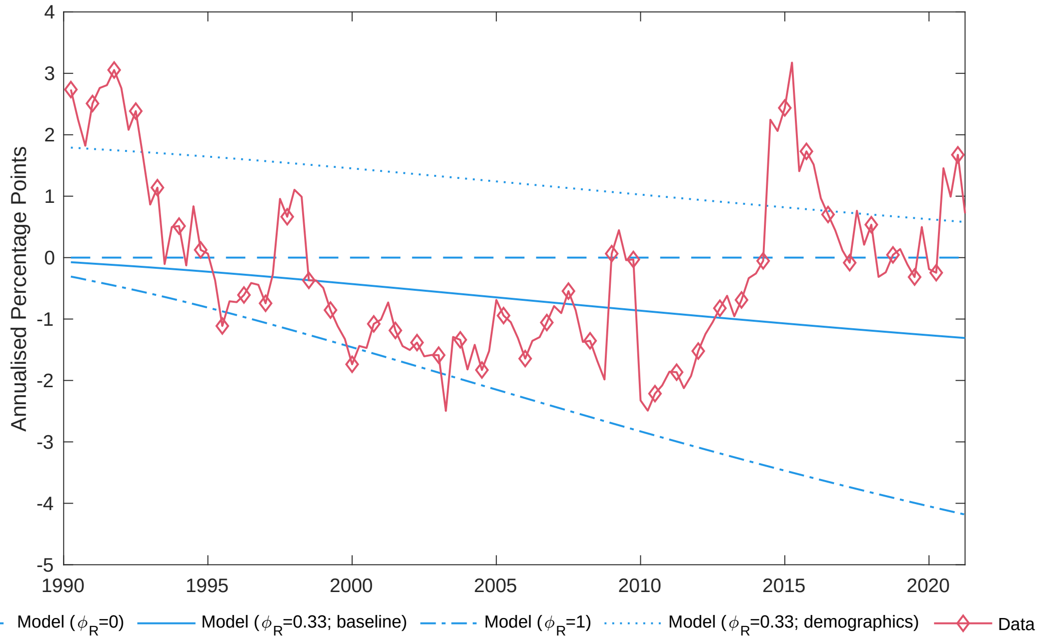
cline in interest rates from the demographic trend exerts upward pressure on the MPC. Though, we can see in Figure 3 that this force is weaker for the elder households (hence the elder-young MPC ratio declines), because of the increase in life expectancy and therefore the need to save. Meanwhile, real wages in the economy increase since the marginal product of labour is higher from the increased capital accumulation.[15]

Japan has seen a secular decline in real interest rates since the collapse in asset prices in the early 1990s. Figure 4 plots the short-term real interest rate constructed with the "Overnight Call Money/Interbank Rate" and inflation (using the GDP deflator) in red compared to the real interest rate from the model (Simulation 1; baseline) in blue between 1990 and 2021. Recall that in our baseline simulation we only feed the model a decline in the population growth rate and an increase in life expectancy – the main drivers outlined in Carvalho, Ferrero, and Nechio (2016), and significant factors in the analysis of Cesa-Bianchi, Harrison, and Sajedi (2023). Thus, our baseline simulation suggests that the demographic decline accounts for an approximate 2 percentage point decline in the real interest rate between 1990 and 2021.

In Figure 5, we plot Japanese inflation in red and model-implied inflation in blue. Here we conduct several counterfactuals to understand how demographics and the

15. We run a robustness check based on Simulation 1 in which agents learn that their population forecasts were overly pessimistic. See Appendix D for details.

Figure 5: Model vs data: Japanese inflation

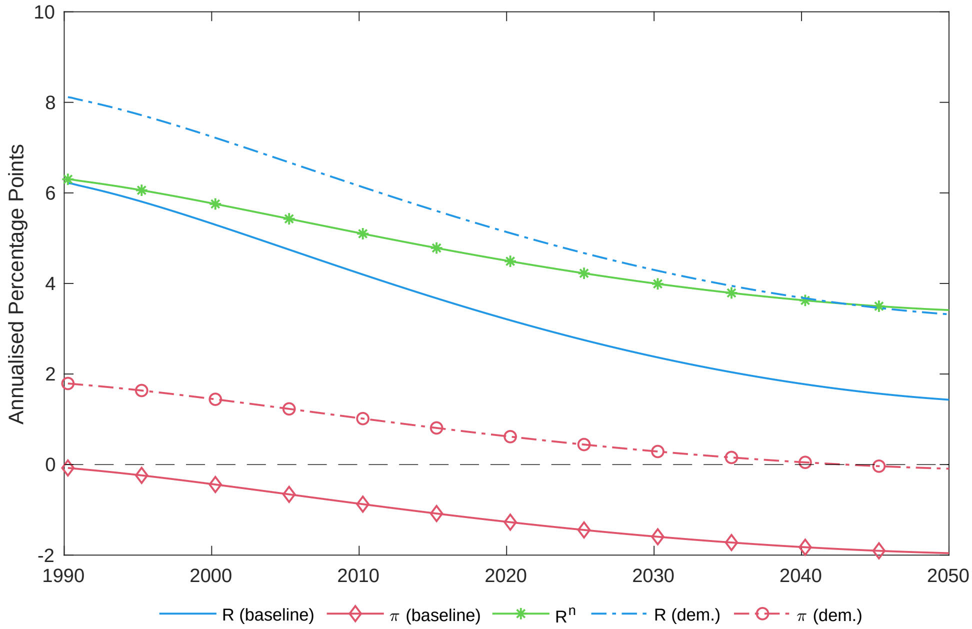


Note: Figure plots quarterly annualised net inflation rates for the model (blue) and data (red diamond). Data used is Japanese inflation constructed from the GDP deflator. The dotted line is model implied inflation when the central bank accounts for Japan's demographic transition. Data sourced from FRED, IMF International Financial Statistics, and the OECD.

central bank's accounting for demographics affects inflation. The solid blue line is the path of inflation (as in Figure 3) from the baseline simulation, where the central bank places some weight, or belief, that the equilibrium natural interest rate is constant.[16] The dashed blue line shows inflation when the central bank places no weight on a constant equilibrium real interest rate – in other words, $\phi_R = 0$ in the monetary policy rule (35). As such, the central bank sets nominal interest rates such that it tracks the flexible price/natural interest rate, and so the inflation gap is always closed. Conversely, the dashed-dotted blue line shows inflation when the central bank sets nominal interest rates by naively believing that the natural interest rate is constant ($\phi_R = 1$). Importantly, both in the baseline simulation and this last simulation with a naive central bank rule (with $\phi_R = 1$), the central bank's reference constant equilibrium real interest rate is set to its initial steady state level in 1990 – just above 6% per annum. In other words, $\bar{R}$ does not take into account the Japanese demographic transition, and is set to its initial steady state value and not its terminal steady state value. However, the dotted blue line shows the path of inflation when the central bank places some weight on a constant equilibrium real interest rate, but at its terminal steady state value – that is,

16. Recall that $\phi_R = 0.33$ as shown in Table 1.

Figure 6: Accounting for demographics: Different interest rate and inflation paths



Note: The solid blue, red diamond, and green starred lines are the net annualised nominal interest rate, inflation rate, and flexible price/natural interest rate, respectively, from Simulation 1 (baseline) of the model whereby the central bank attaches weight ϕ_R to a constant natural interest rate without taking into account demographics (see Equation (35)). The dashed-dotted blue and red lines are the net annualised nominal interest rate and inflation rate, respectively, when the central bank does take into account demographics in its constant natural interest rate. The weight $\phi_R = 0.33$ is identical across both simulations.

the steady state interest rate that takes into account the demographic transition. When the central bank does this, the path of inflation is positive as opposed to deflationary in the naive case when the central bank does not account for demographics.

To further visualise this, in Figure 6 we plot nominal interest rates (blue solid), inflation (red diamond), and the flexible price or natural interest rate (green starred). The solid lines show the time paths for the nominal interest rate and inflation from the baseline simulation where the central bank places some weight on the equilibrium interest rate being constant without taking into account demographics. It should be clear why inflation trended below steady state in Figure 5: due to Japan's demographic transition, the equilibrium real interest rate decreases from just above 6% in 1990 to approximately 3% per annum by 2050, as can be seen by the path of the natural interest rate R^n. Despite this, the central bank sets nominal interest rates using a level of $\bar{R}$ that is consistent with the initial steady state value of the natural interest rate of 6% per annum. In other words, the central bank – by not accounting for the demographic transition – is setting an overly tight monetary policy resulting in deflation in the economy.

Contrast this for the case where the central bank does account for the demographic transition – depicted by the dashed-dotted lines in Figure 6. Now, because the central bank sets nominal interest rates with $\bar{R}$ consistent with the terminal steady state natural interest rate level of approximately 3% per annum, the economy sees inflation instead of deflation during the simulation period.

In other words, our simulation suggests that higher nominal interest rates coincide with higher inflation; counter-intuitive to standard macroeconomic analysis of monetary policy and inflation. However, recall that our agents operate in a perfect foresight or deterministic environment, and so one can interpret the mechanism for our results as neo-Fisherian as explained in García-Schmidt and Woodford (2019), Bilbiie (2022), and Cochrane (2024).[17]

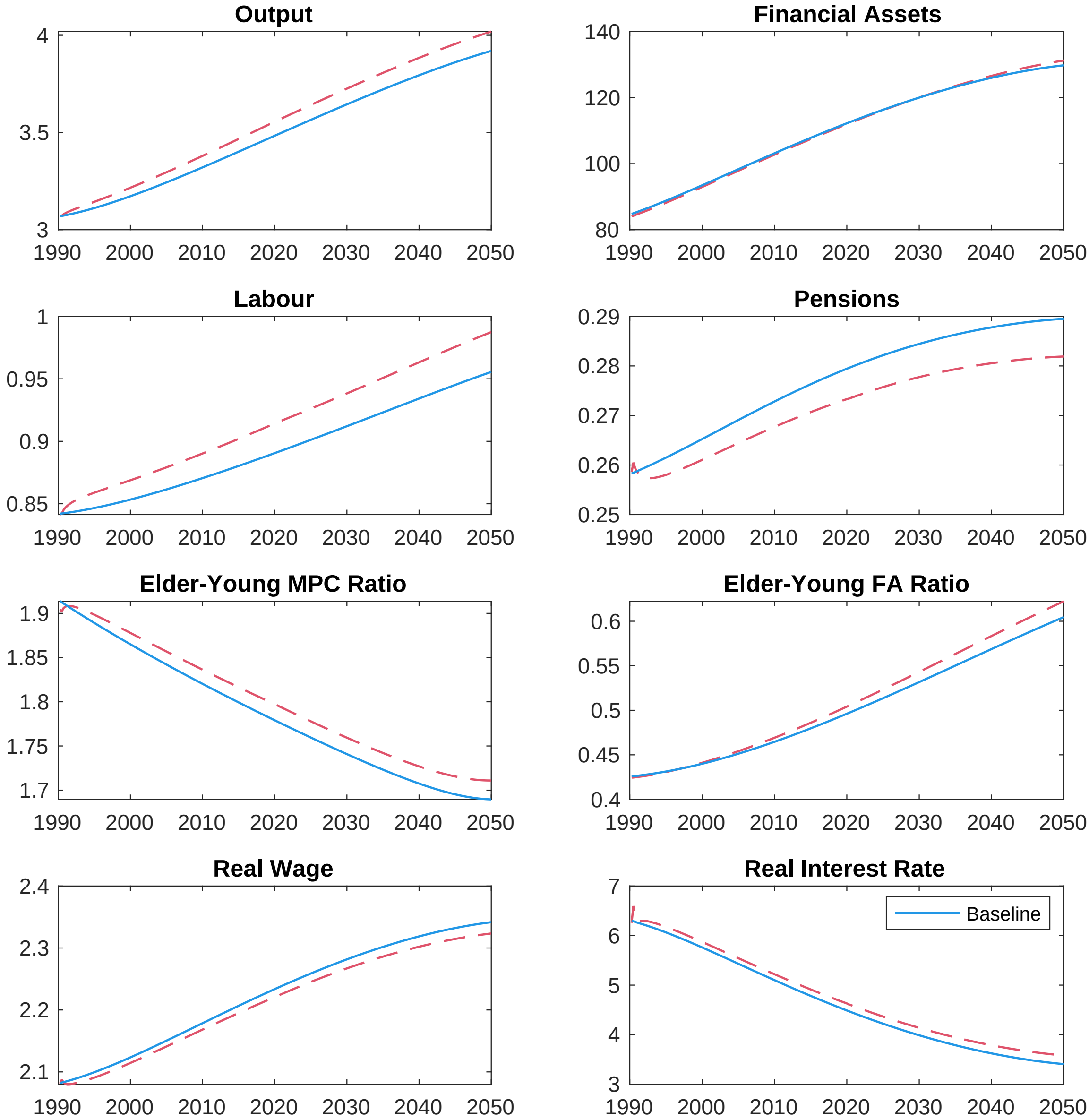
Simulation 2: An increase in elder worker productivity. Now, we add a third demographic shock observed between 1990 and 2017: the relative productivity of elderly workers to young workers (0.75 to 0.85 in the calibration as in Table 1). The time paths for key variables are plotted in Figure 7. The shock's effect on financial assets in the economy are small but important for the effect on the real interest rate. The ratio of financial assets held by the elderly increases as a result of higher productivity for elder workers. There are two channels in place: First, through the income effect of higher real wages, elder workers can accumulate more assets. Second, young workers hold less assets because saving for old age is less rewarding (there is a substitution effect). The young workers holding less assets leads to higher interest rates relative to the baseline case.

Since elderly workers enjoy higher productivity, labour supply increases compared to the baseline case, and thus output expands. The MPC ratio for elderly and young workers is higher than in the baseline case because the elder workers have higher wage income and are willing to consume more output. However, relative to the baseline simulation slightly higher real interest rate imply that in this simulation there is less capital in the economy, and therefore a lower overall marginal product of labour (despite the increase in productivity for elder workers). Hence, the headline aggregate real wage is lower than the baseline case. Since real wages determine pension payments, the pensions are also lower than the baseline case.

To emphasise the effect on interest rates, a decrease in population growth and an increase in life expectancy (baseline) decrease the real interest rate, as we see in the data. An increase in elder worker productivity puts upward pressure on interest rates, but

17. We omit discussion on long-run neo-Fisherian stability and money neutrality in this paper, and refer interested readers to the aforementioned pieces by Garcia-Schmidt and Woodford, Bilbiie, and Cochrane, as well as Cochrane (2016).

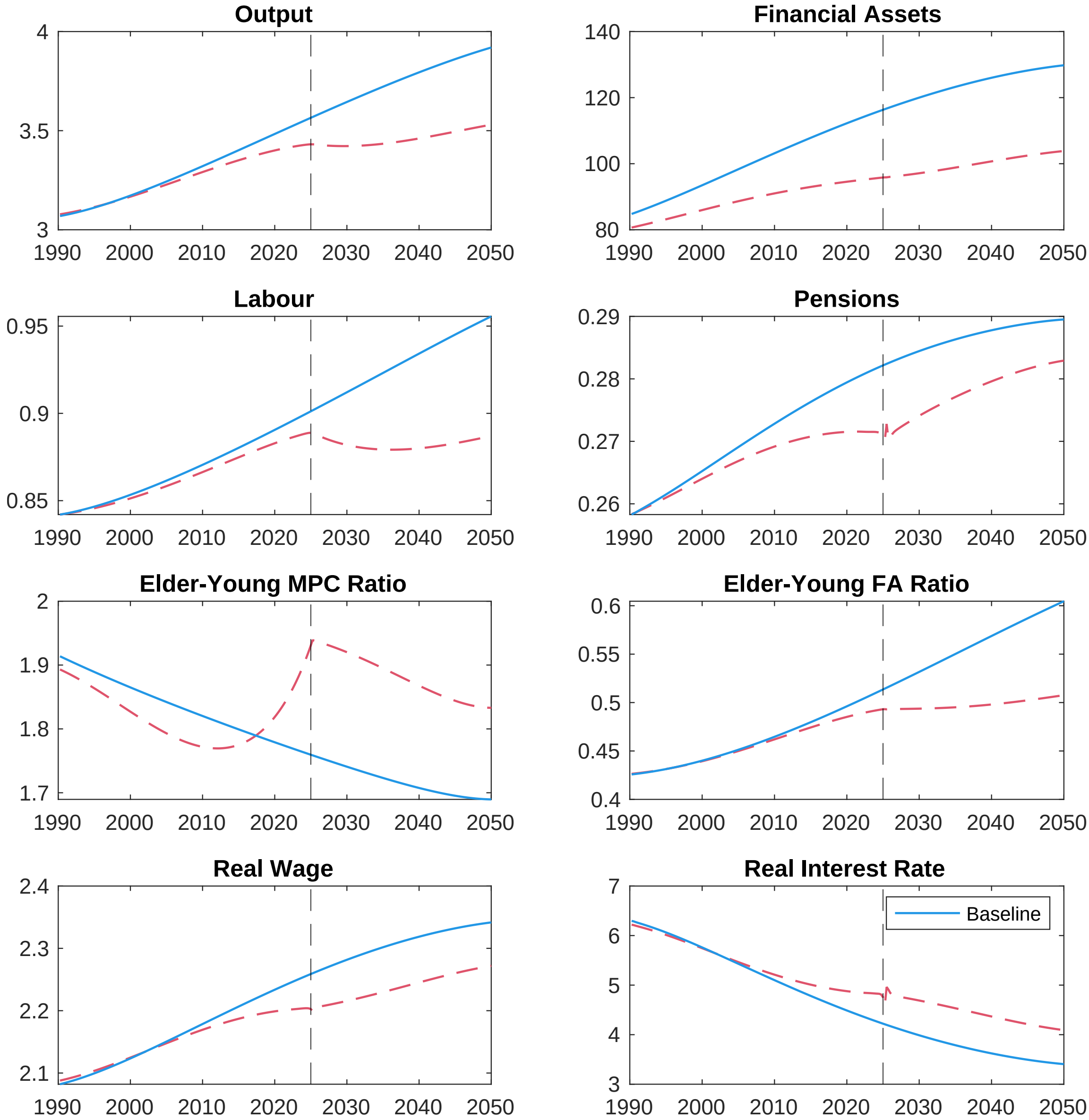
Figure 7: Simulation 2 (increase in elder worker productivity)



Note: Figure plots response of variables as n transitions from 0.0011 to -0.0014, γ transitions from 0.9815 to 0.9860, and ς transitions from 0.75 to 0.85. Interest rate is expressed in net annualised percentage points.

not nearly enough to counteract the first two effects. As Japan has experienced all three of these shocks in the past few decades, we can deduce that the decrease in population growth and the increase in life expectancy played a bigger role in the determination of real interest rates. However, policy to increase the productivity of elder workers could alleviate the downward secular trend in real interest rates.

Figure 8: Simulation 3 (increase in retirement age)



Note: Figure plots response of variables as n transitions from 0.0011 to −0.0014 and γ transitions from 0.9815 to 0.9860. ω changes from 0.9944 to 0.9950 in 2025. Interest rate is expressed in net annualised percentage points.

Simulation 3: An increase in the retirement age. Here, we introduce a policy counterfactual: an increase in the retirement age from 60 to 70 years of age. Since we assume that elder households keep working, but with lower productivity, we can also rephrase it as a shock for which workers are productive for longer. Simulation results for the time path of variables are displayed in Figure 8.

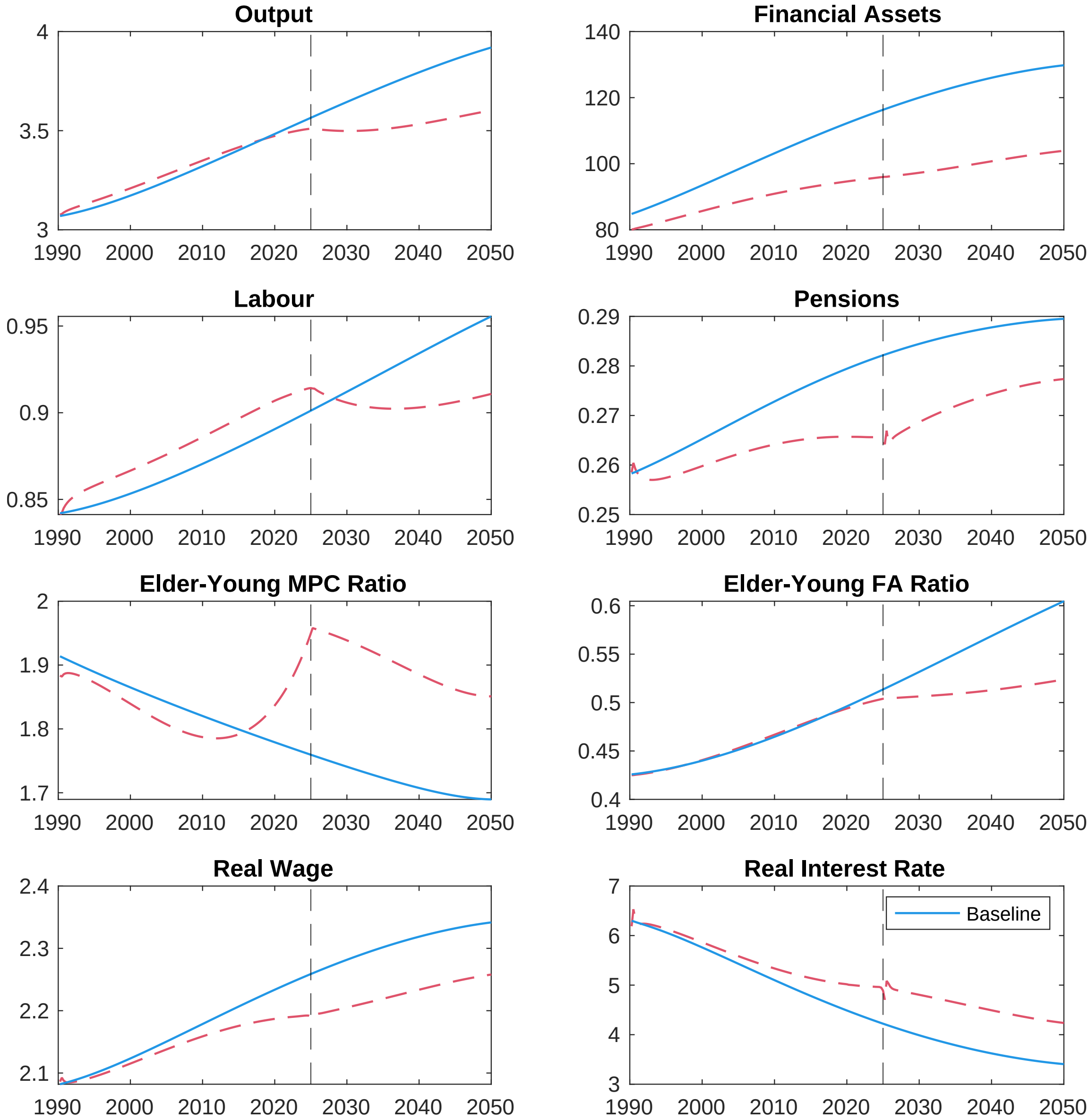
In the simulation, we assume that agents know from the start that the retirement age is going to increase in the year 2025, where the vertical dashed line is. The increase in retirement age is similar to the previous shock, as both increase the productivity of the marginal elder worker. With an increase in the retirement age, the marginal elder worker obtains the same productivity as a young worker. However, with the increase in productivity of the elder worker, all elder workers obtain a higher productivity but not as high as the young.

With an increase in the retirement age, the increase in financial assets is smaller than in the baseline case because agents need to save less. This is because, all else being equal, a deferral in the retirement age also coincides with a decline in the survival probability of an elderly worker – in other words, γ contemporaneously declines for a given life expectancy. There is also a smaller increase in the financial assets ratio of the elderly because there are fewer elder workers in the economy compared to the baseline simulation. The smaller asset accumulation through a lower supply of savings results into a higher interest rate and a lower level of capital in the economy. Hence, the marginal productivity of labour is lower, which causes real wages and labour to increase by much less than in the baseline scenario. Output follows the labour path closely and also hardly increases after the increase in retirement age. The ratio of the elderly-young MPC is higher after the shock materialises because agents are willing to increase their consumption in anticipation of remaining productive for longer during their youth.

All in all, an increase in the retirement age, which makes workers more productive for longer, puts an upward pressure on real interest rates because the level of capital is lower than in the baseline case. So, even though a decrease in population growth and an increase in life expectancy (baseline) decreases real interest rates in the economy, making the marginal elder worker more productive alleviates the downward pressure on interest rates, as we saw in the previous exercise. Comparing the two exercises, we can conclude that making the marginal elder worker as productive as the young (increasing retirement age) has a bigger alleviating effect than making all elder workers more productive (increasing relative elder worker productivity). The difference in capital accumulation between these two scenarios drives these results. With an increase in the retirement age, the financial assets are a lot less than under the baseline case, whereas for the increase in elder worker productivity the effect on capital accumulation is significantly smaller.

Simulation 4: Increase in retirement age and elder worker productivity. In this last exercise, we combine the previous two simulations: as in, elderly worker productivity increase (increase in ς) and the government defers the retirement age to 70 years of

Figure 9: Simulation 4 (increase in retirement age and elder worker productivity)



Note: Figure plots response of variables as n transitions from 0.0011 to −0.0014, γ transitions from 0.9815 to 0.9860, and ς_t transitions from 0.75 to 0.85. ω changes from 0.9944 to 0.9950 in 2025. Interest rate is expressed in net annualised percentage points.

age in the year 2025. Results are shown in Figure 9.

As discussed, the shocks are similar in the sense that they both increase the productivity of the marginal elder worker. Combining the shocks amplifies the effect on real interest rates and alleviates its secular decline. For the rest of the economy, the effect of the increase in retirement age is bigger than the increase in elder worker productivity, which makes sense when looking at the previous two exercises and the time path of model variables.

Thus, a decrease in population growth rate together with an increase in life expectancy (baseline) indeed decreases real interest rates, as observed in Japan in the past few decades. Our results show that the increased productivity of the elderly, marginal or as a whole, most likely did alleviate the decline, and that policy to make elder workers even more productive is beneficial to steeper declines in real interest rates.

3.2 Demographics and Fiscal Stimulus

In this section, we look at the implications of a demographic transition on the effects of a fiscal stimulus, namely a tax cut. This is motivated by a recent policy of the Japanese government that introduced a fiscal stimulus package comprised of approximately ¥7 trillion worth of tax cuts.[18]

We adjust the fiscal policy block of the model economy such that tax receipts of the government are a fraction of GDP instead of being a residual as in the baseline case.[19] By implication, government debt is determined residually via the government budget constraint. The tax rule takes the following form:

$$T_t = \psi_t Y_t, \tag{38}$$

where ψ_t denotes an exogenous and time-varying debt-to-GDP ratio.

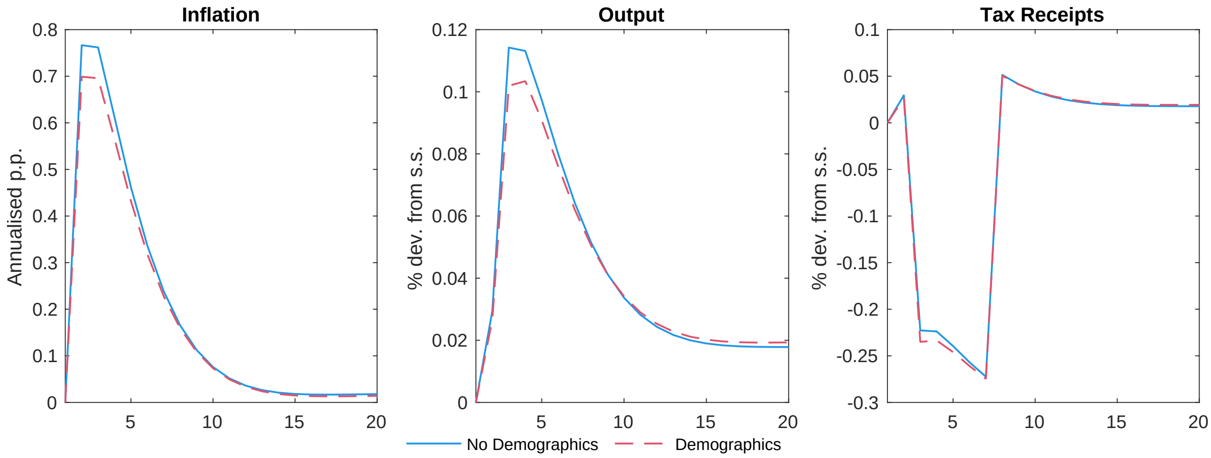
Under rule (38), fiscal policy is "active" in the sense of Leeper (1991). Hence, to ensure existence of a unique solution and local stability, we assume that the central bank reacts to deviations of inflation less than one-to-one ($\phi_\pi = 0.8$).

To study the effects of a fiscal stimulus, we assume that the tax-to-GDP ratio ψ_t drops by 10 basis points from 29.71% to 29.61% for four consecutive quarters. This tax cut occurs in period 3 but are announced in a period before. We simulate the model for 20 periods with a demographic transition, assuming a population decline of 0.8% per year, and without. The results of the simulation are presented in Figure 10.

We observe that the fiscal stimulus is less inflationary under the negative population growth case. This is due to the fact that following a tax cut, the government increases borrowing which is not repaid by tax adjustments in the future.[20] Since the government does not implement tax adjustments to repay additionally accumulated debt, it has to be inflation-eroded, as is standard under the FTPL in a sticky-price environment. Under a declining population, there is less debt per-capita to be eroded by inflation –

18. See the following links for further information:
https://www.ft.com/content/0f1deddc-0862-45d1-b754-d594c79d5f18
https://www.japantimes.co.jp/business/2024/05/31/june-tax-cut/.
19. To be clear: (32) is replaced by the tax rule (38).
20. Recall that Ricardian equivalence does not hold in this model environment.

Figure 10: Effects of a temporary tax cut



Note: Model is simulated with active fiscal policy and passive monetary policy ($\phi_\pi = 0.8$) conditional on a 10 basis point decline in the tax-to-GDP ratio. "No Demographics" indicates that there is no population growth ($n = 0$), whereas "Demographics" indicates that population growth is -0.8% per annum ($n = -0.002$). Inflation is annualised, and Output and Tax Receipts are expressed in terms of percent deviations from deterministic steady states.

hence the lower inflation.

Before concluding this exercise, it is worth contextualising our findings. As Figure 10 shows, the tax cuts introduced in our model are self-financed in line with the complementary channels discussed in Angeletos, Lian, and Wolf (2023): i) the tax base channel and ii) the inflation/debt erosion channel, the latter of which is of primary focus of the FTPL literature. But the self-financing in our model is different from the literature on the "free lunch" or "$r < g$" condition (Blanchard, 2019; Mehrotra and Sergeyev, 2021; Reis, 2022). Instead, the self-financing of debt in our model is driven by a Keynesian boom.

4 Conclusion

This paper uses an OTANK model based on Gertler (1999), complete with social security and variable labour, and Japanese demographic and wage data to assess the effects of: i) a decrease in the productivity gap between young and old workers; and, ii) deferring the effective age of retirement. We use our OTANK model to conduct policy counterfactuals to investigate the extent to which higher productivity of the marginal elderly worker alleviates the downward pressure on real interest rates due to demographic ageing.

We find that since the collapse of the asset price bubble in the early 1990s, average wage earning profiles across the life-cycle for Japanese workers has flattened. In other words, the wages gap across different aged cohorts since the early 1990s has shrunk. If one assumes that wages accurately reflect productivity, then the flattening of earnings profiles across the life-cycle can be interpreted as a convergence of productivity across different aged workers. Through the lens of our model, the increase in the relative productivity of elderly workers exerts upward pressure on the interest rate. As elderly workers become more productive, the earnings potential of an individual worker undergo a smaller decline when they transition from young to old. As such, the incentive to save during their youth is diminished, decreasing savings and the capital stock, and therefore increasing interest rates. We find similar and even stronger effects when we increase the retirement age.

Despite the increase in the relative productivity of the elderly between 1990 and 2017, when simulating the model calibrated to Japanese data, this study finds downward pressure on interest rates stemming from the ageing society overpower the upward pressures coming from changes in relative productivity. This is consistent with the real world experience in Japan (Lise et al., 2014) and studies on the secular decline in natural interest rates (Bailey et al., 2022; Cesa-Bianchi, Harrison, and Sajedi, 2023). Additionally, we find that if the central bank naively sets nominal interest rates without accounting for these demographic transitions (and the consequent fall in natural interest rates), then it may induce deflation as its monetary policy is too strict.

The findings in this paper have strong implications for policy makers, particularly in Japan which faces challenging demographic transitions and ballooning public pension expenses. Unsurprisingly, deferring the age of effective retirement (from 60 to 70 in our counterfactual simulations) is only a temporary fix. While it sets an asymptotic old age dependency ratio of 0.40, an improvement over the baseline rate of 0.58, it is relatively ineffective in combating the deflationary pressures discussed here and in other studies such as Carvalho, Ferrero, and Nechio (2016). A policy mix of deferring the effective age of retirement and encouraging higher relative productivity of elderly workers is a first best option.

Our findings suggest that decreasing the dependency ratio, and reducing asset accumulation in the economy, could alleviate the downward pressure on both interest rates and inflation. Furthermore, we find that negative population growth rates have important implications for fiscal stimulus. We verify the mechanics highlighted in Mian, Straub, and Sufi (2022) and Angeletos, Lian, and Wolf (2023): debt is self-financed and stimulates real output and inflation in an environment with sticky-prices and a failure of Ricardian equivalence – as in our model. However, we show that fiscal stimulus efficacy is lower as the economy undergoes population decline.

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